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Low-temperature magnetization in nanofilms

Vjekoslav Sajfert [a,*], Dušan Popov [b], Bratislav Tošić [c]

[a] *Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia*
[b] *Universitatea "Politechnica", Timisoara, Romania*
[c] *Vojvodina Academy of Science and Arts, Novi Sad, Serbia*

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ABSTRACT

The low-temperature magnetization of a film was analyzed by the use of exact Bose representation of spin operators that does not suffer from the presence of unphysical states. The magnetization of thin films has exponentially small temperature correctness, so that Dyson's proof about exponentially small correction coming from two bosons at ideal lattice point cannot be used in film analyses. The main conclusions of this work are that magnetic lattice of a thin film is more rigid than the macroscopic lattice and that the autoreduction process (the three layer film divides into two layer subfilms) takes place in the film.

0. Introduction

The problem of operators with mixed kinematics arose 80 years ago. Bosons and fermions have clean kinematics while spin operators [1], Pauli operators and quasi-Pauli operators [1–6] have mixed kinematics (their kinematics is a combination of Bose and Fermi kinematics). Mixed kinematics leads to statistical problems (the calculation of mean values of operator products) and correspondingly to thermodynamical problems. Since thermodynamics deals with measurable quantities, correct evaluation of statistical means is necessary.

At first the problem of mixed kinematics and corresponding mixed statistics arose in quantum theory of magnetism. All attempts to solve this problem were based on the idea of finding an adequate boson representation of spin operators which would give correct statistical means. Following the appearance of lasers the interaction between optical excitations became topical. The original operators in the Hamiltonian of optical excitations (Pauli and quasi-Pauli operators) had mixed kinematics and statistics too, and boson representations of these operators were necessary for the explanation of nonlinear optical effects, such as energy shift in optical spectra and two- and three-photon absorption [7,8].

A short review of attempts concerning the problem of mixed kinematics is presented in the first section. The second section is dealing with the problem of the low-temperature magnetization in thin films and the results of the second section are summarized in the conclusion.

* Corresponding author. Tel.: +381 23 550 515; fax: +381 23 550 520.
E-mail addresses: sajfertv@ptt.rs (V. Sajfert), dusan_popov@yahoo.co.uk (D. Popov), btosic@yahoo.com (B. Tošić).

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1. A short review of problems concerned with mixed kinematics

The problem of the low-temperature magnetization of Heisenberg ferromagnet was started by Bloch [9] who approximated Heisenberg isotropic spin Hamiltonian with quadratic boson form. The Hamiltonian of Heisenberg ferromagnet is given by:

$$H = SJ_0 \sum_{\vec{n}} \left(S - S^z_{\vec{n}}\right) - \frac{1}{2}\sum_{\vec{n},\vec{m}} I_{\vec{n},\vec{m}} S^-_{\vec{n}} S^+_{\vec{n}} - \frac{1}{2}\sum_{\vec{n},\vec{m}} I_{\vec{n},\vec{m}} \left(S - S^z_{\vec{n}}\right)\left(S - S^z_{\vec{m}}\right) \tag{1.1}$$

where $S^+ = S^x + \mathrm{i}S^y$, $S^+ = S^x + \mathrm{i}S^y$ are spin operators, are exchange integrals and $J_0 = \sum_{\vec{l}} I_{\vec{l}}$. The spin operators obey the following commutation relations:

$$\left[S^+, S^-\right] = 2S^z; \qquad \left[S^+, S^z\right] = -S^+; \qquad \left[S^-, S^z\right] = S^-. \tag{1.2}$$

Bloch used boson representation of spin operators:

$$S^+_{\vec{n}} = \sqrt{2S} B_{\vec{n}}; \qquad S^-_{\vec{n}} = \sqrt{2S} B^+_{\vec{n}}; \qquad S - S^z_{\vec{n}} = B^+_{\vec{n}} B_{\vec{n}}. \tag{1.3}$$

and obtained equivalent boson Hamiltonian:

$$H = SJ_0 \sum_{\vec{n}} B^+_{\vec{n}} B_{\vec{n}} - S \sum_{\vec{n},\vec{m}} I_{\vec{n},\vec{m}} B^+_{\vec{n}} B_{\vec{m}}. \tag{1.4}$$

By diagonalisation of Hamiltonian (1.4), he found the dispersion law $E = S\left(J_0 - J_{\vec{k}}\right)$ where $J_{\vec{k}} = \sum_{\vec{m}} I_{0\vec{m}} \mathrm{e}^{i\vec{k}\vec{m}}$ and using it he obtained the following expression for magnetization:

$$\sigma = \frac{\langle S^z \rangle}{S} = 1 - \frac{1}{S}\zeta_{3/2}\tau^{3/2}; \quad \tau = \frac{\Theta}{2\pi I}; \Theta = k_B T \tag{1.5}$$

which is called "Bloch's 3/2 law". The Riemann's ζ function is given by:

$$\zeta_p = \sum_{n=1}^{\infty} \frac{1}{n^p}. \tag{1.6}$$

The next step in the theory of low-temperature magnetization was done by Holstein and Primakoff [4], who used the following boson representation for spin operators:

$$S^+ = \sqrt{2S}\sqrt{1 - \frac{B^+B}{2S}}B; \qquad S^- = \sqrt{2S}B^+\sqrt{1 - \frac{B^+B}{2S}}$$
$$S^z = S - B^+B. \tag{1.7}$$

Using this representation of spin operators, they obtained the expression for low-temperature magnetization:

$$\sigma = \sigma_h + \sigma_{ah} \tag{1.8}$$

where

$$\sigma_h = 1 - 2\zeta_{3/2}\tau^{3/2} - \frac{3\pi}{2}\zeta_{5/2}\tau^{5/2} - \frac{33\pi^2}{16}\zeta_{7/2}\tau^{7/2} + O\left(\tau^{9/2}\right), \tag{1.9}$$

and

$$\sigma_{ah} = -6\pi\zeta_{3/2}\zeta_{5/2}\tau^4. \tag{1.10}$$

The terms in formula (1.9) are coming from expanding the dispersion law in powers of magnitude $\left|\vec{k}\right|$ of $\vec{k}$ up to k^6 while anharmonic correction is the consequence of the interaction between spin waves.

The result (1.8)–(1.10) was correct but the boson representation of spin operators, which is correct only if number of bosons at one lattice point is maximally equal to $2S$ arose discussions concerning the contributions of so-called "unphysical states", i.e. the states with more than $2S$ boson per lattice point.

The completed theory of the low-temperature magnetization was given by Dyson [10,11]. He proved that the states with more than one boson at one lattice point involve exponentially small contributions in the low-temperature magnetization. Using this proof, he obtained that Holstein and Primakoff's result was correct.

Finally, the discussions about "unphysical states" were put *ad acta* by the appearance of Agranovich–Toshich [3] boson representation of spin operators for spin operators $S = 1/2$:

$$S^{+} = \left[\sum_{\nu=0}^{\infty} \frac{(-2)^{\nu}}{(1+\nu)!} B^{+\nu} B^{\nu}\right]^{\frac{1}{2}} B; \qquad S^{-} = B^{+}\left[\sum_{\nu=0}^{\infty} \frac{(-2)^{\nu}}{(1+\nu)!} B^{+\nu} B^{\nu}\right]^{\frac{1}{2}};$$
$$\frac{1}{2} - S^{z} = \sum_{\nu=0}^{\infty} \frac{(-2)^{\nu}}{(1+\nu)!} \left(B^{+}\right)^{\nu+1} B^{\nu+1}. \tag{1.11}$$

and by exact boson representation of quasi-Pauli operators [5]. It should be noticed that boson infinite series (1.11) satisfy commutation rules (1.2) for an arbitrary number of bosons. Using the representation (1.11), the exact results for the low-temperature ordering parameter of antiferromagnet was determined [12] and the exact result for magnetization in case $S > 1/2$ was found in [6]. The correct result of Holstein–Primakoff's formula (1.8) was obtained in [13].

Using the formula (1.11) Agranovich and Toshich predicted the possibility of Bose condensation of Frenkel excitons. The similar idea was applied to ferromagnet in strong magnetic field $\mu_B H >> I$ [14] and the Bose condensation of spin waves was predicted. This prediction has been recently proved experimentally for YIG ferromagnet films in [15].

The exact Bose representation of spin operators (1.11) will be applied for the calculation of magnetization of ultrathin ferromagnet films with $S = 1/2$.

Ending this review of the low-temperature problems in determining magnetization, we quote the leading correction which is given by Bloch's approach in three layer film. This correction is proportional to $\tau Z_1\left(\frac{1}{4\pi\tau}\right)$, where $Z_p(q) = \sum_{n=1}^{\infty} \frac{e^{-nq}}{n^p}$ and it is clear that it is exponentially small.

2. Low-temperature magnetization in three layer magnetic films with spin $S = 1/2$

The interest in magnetic problems has increased recently [16–24], but nowhere has the problem of broken symmetry magnetic structures been in the foreground.

We shall analyze the three layer magnetic film with spin $S = 1/2$ cut off in z direction from simple cubic ideal structure. The more compact and clearer formulae will be obtained if we substitute spin operators with Pauli operators in accordance with the following formulae:

$$S^{+} = P, \qquad S^{-} = P^{+}; \qquad \frac{1}{2} - S^{z} = P^{+}P; \qquad \left[P_{\vec{n}}, P^{+}_{\vec{m}}\right] = 1 - 2P^{+}_{\vec{n}}P_{\vec{n}}; \qquad \left(P^{+}_{\vec{n}}\right)^{2} = (P_{\vec{n}})^{2} = 0 \tag{2.1}$$

Using (2.1) and the nearest neighbour approximation for a simple cubic structure, we obtain the Hamiltonian of three layer film in the following form:

$$\begin{aligned} H =\ & \frac{1}{2}\sum_{\vec{n}}\left(I_{\vec{n},\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}} + I_{\vec{n},\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}} + I_{\vec{n},\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}\right)P^{+}_{\vec{n}}P_{\vec{n}} - \frac{1}{2}\sum_{\vec{n}} P^{+}_{\vec{n}}\left(I_{\vec{n},\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P_{\vec{n}-\vec{a}}\right. \\ & \left. + I_{\vec{n},\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P_{\vec{n}-\vec{a}} + I_{\vec{n},\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P_{\vec{n}-\vec{a}}\right) - \frac{1}{2}\sum_{\vec{n}} P^{+}_{\vec{n}}P_{\vec{n}}\left(I_{\vec{n},\vec{n}+\vec{a}}P^{+}_{\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P^{+}_{\vec{n}-\vec{a}}P_{\vec{n}-\vec{a}}\right. \\ & \left. + I_{\vec{n},\vec{n}+\vec{a}}P^{+}_{\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P^{+}_{\vec{n}-\vec{a}}P_{\vec{n}-\vec{a}} + I_{\vec{n},\vec{n}+\vec{a}}P^{+}_{\vec{n}+\vec{a}}P_{\vec{n}+\vec{a}} + I_{\vec{n},\vec{n}-\vec{a}}P^{+}_{\vec{n}+\vec{a}}P_{\vec{n}-\vec{a}}\right) \end{aligned} \tag{2.2}$$

where $\vec{a}$ connects the nearest neighbours.

In formula (2.2) the boundary conditions must be taken into account on the surfaces 0 and 2. It means that boundary conditions are:

$$I_{n_x,n_y,0;n_x,n_y,-1} = I_{n_x,n_y,N_z;n_x,n_y,N_z+1} = 0. \tag{2.3}$$

The other exchange integrals will be denoted by I.

The analysis will be carried out by means of the Pauli–Green function

$$\Gamma_{\vec{n},\vec{m}}(t) = \theta(t)\left\langle\left[P_{\vec{n}}(t), P^{+}_{\vec{m}}(0)\right]\right\rangle \equiv \langle\langle P_{\vec{n}}(t)\,|P^{+}_{\vec{m}}(0)\rangle\rangle, \tag{2.4}$$

where $\theta(t)$ is a Heaviside step-function. Differentiating $\Gamma_{\vec{n},\vec{m}}(t)$ with respect to t and taking into account that $\dot{P}_{\vec{n}} = \frac{1}{i\hbar}[P_{\vec{n}}, H]$, we obtain the equation for Pauli–Green function $\Gamma_{\vec{n},\vec{m}}(t)$:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\langle\langle P_{\vec{n}}(t)\,|P^{+}_{\vec{m}}(0)\rangle\rangle = i\hbar\delta(t)\,\delta_{\vec{n},\vec{m}}\left(1 - 2\langle P^{+}_{\vec{n}}P_{\vec{n}}\rangle\right) + \langle\langle[P_{\vec{n}}, H]_{t}\,|P^{+}_{\vec{m}}(0)\rangle\rangle. \tag{2.5}$$

Further steps in the analysis are the following:

1. In accordance with (1.11) and (2.1), Pauli operators will be substituted with:

$$P_{\vec{n}} = B_{\vec{n}} - B^+_{\vec{n}} B_{\vec{n}} B_{\vec{n}}; \qquad P^+_{\vec{n}} = B^+_{\vec{n}} - B^+_{\vec{n}} B^+_{\vec{n}} B_{\vec{n}};$$
$$P^+_{\vec{n}} P_{\vec{n}} = B^+_{\vec{n}} B_{\vec{n}} - \left(B^+_{\vec{n}}\right)^2 B^2_{\vec{n}} \tag{2.6}$$

in the Green function $\langle\langle [P_{\vec{n}}, H]_t \,|P^+_{\vec{m}}(0)\rangle\rangle$ and in correlation function, while in the higher order Green functions of the type $\langle\langle P^+_{\vec{a}}(t)\, P_{\vec{a}}(t)\, P_{\vec{b}}(t)\,|P^+_{\vec{c}}(0)\rangle\rangle$ will be substituted with Bose operators, i.e. it will be taken that $P = B$.

2. Applying Wick's theorem [25], we can write down:

$$\begin{aligned}\langle\langle P_{\vec{n}}(t)\,|P^+_{\vec{m}}(0)\rangle\rangle &\approx \langle\langle \left(B_{\vec{n}}(t) - B^+_{\vec{n}}(t)\,B_{\vec{n}}(t)\,B_{\vec{n}}(t)\right) \mid \left(B^+_{\vec{m}}(0) - B^+_{\vec{m}}(0)\,B^+_{\vec{m}}(0)\,B_{\vec{n}}(0)\right)\rangle\rangle \\ &\approx \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle - \langle\langle B^+_{\vec{n}}(t)\,B_{\vec{n}}(t)\,B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle - \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\,B^+_{\vec{m}}(0)\,B_{\vec{n}}(0)\rangle\rangle \\ &\approx \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle - 2\langle B^+_{\vec{n}} B_{\vec{n}}\rangle^{(0)} \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle - 2\langle B^+_{\vec{m}} B_{\vec{m}}\rangle^{(0)} \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle.\end{aligned} \tag{2.7}$$

It should be noticed that the higher order Green functions of the type $\langle\langle B^+BB|B^+B^+B\rangle\rangle$ are rejected, since they are proportional to the square of boson concentration $\langle B^+B\rangle$.

Inserting the formulae obtained by the application of Wick's theorem for bosons into (2.5), applying the boundary conditions (2.3), and using the transformation for the boson Green function $G_{\vec{n},\vec{m}}(t) = \langle\langle B_{\vec{n}}(t)\,|B^+_{\vec{m}}(0)\rangle\rangle$ in which translatory invariance in x, y plane is taken into account:

$$\begin{aligned}G_{n_x,n_y,n_z;m_x,m_y,m_z}(t) &= \langle\langle B_{n_x,n_y,n_z}(t)\,|B^+_{m_x,m_y,m_z}(0)\rangle\rangle \\ &= \frac{1}{N_x N_y} \sum_{k_x,k_y} \mathrm{e}^{\mathrm{i}ak_x(n_x-m_x)+\mathrm{i}ak_y(n_y-m_y)} \int_{-\infty}^{+\infty} \mathrm{d}\omega \mathrm{e}^{-\mathrm{i}\omega t} \alpha_{n_z,m_z}\left(k_x,k_y,\omega\right).\end{aligned} \tag{2.8}$$

we obtain the following system of difference equations for the function $\alpha_{n_z,m_z}\left(k_x,k_y,\omega\right)$:

$$n_z = 1 \quad J\left(\alpha_{2,m_z} + \alpha_{0,m_z}\right) + \rho\alpha_{1,m_z} = \frac{i\hbar}{2\pi}\left(1 + 2\langle B^+B\rangle^{(0)}\right)\delta_{1,m_z}; \tag{2.9}$$

$$n_z = 0 \quad (J - \Delta J)\,\alpha_{1,m_z} + (\rho + \Delta\rho)\,\alpha_{0,m_z} = \frac{i\hbar}{2\pi}\left(1 + 2\langle B^+B\rangle^{(0)}\right)\delta_{0,m_z}; \tag{2.10}$$

$$n_z = 2 \quad (J - \Delta J)\,\alpha_{1,m_z} + (\rho + \Delta\rho)\,\alpha_{2,m_z} = \frac{i\hbar}{2\pi}\left(1 + 2\langle B^+B\rangle^{(0)}\right)\delta_{2,m_z}. \tag{2.11}$$

The notations in upper equations are the following:

$$\begin{aligned}&\rho = E - 2I\left(\sin^2\frac{ak_x}{2} + \sin^2\frac{ak_y}{2}\right) - I + \frac{1}{2}I\,(F_1 + 3F_2) - 2I\Phi_2; \\ &\Delta\rho = \frac{1}{2}I - \frac{1}{2}I\,(F_1 + F_2) - \frac{3}{2}I\Phi_2; \qquad \Delta\Phi = \Phi_1 - \Phi_2; \\ &J = \frac{1}{2}I\,(1 - 2F_2); \qquad \Delta J = \frac{1}{4}I\Delta F; \qquad \Delta F = F_1 - F_2\end{aligned} \tag{2.12}$$

where the functions F and Φ are given by:

$$F_\lambda = \frac{1}{N_x N_y}\sum_{k_x,k_y}\frac{1}{\mathrm{e}^{E^{(0)}_{k_x,k_y,\lambda}} - 1} = \tau Z_1\left(\frac{2\lambda-1}{4\pi\tau}\right) + \frac{\pi}{2}\tau^2 Z_2\left(\frac{2\lambda-1}{4\pi\tau}\right) + O\left(\tau^3 Z\right); \quad \frac{2\lambda-1}{2}\frac{I}{\Theta} = \frac{2\lambda-1}{4\pi\tau} \tag{2.13}$$

and

$$\begin{aligned}\Phi_\lambda &= \frac{1}{N_x N_y}\sum_{q_x,q_y}\frac{M\left(k_x,k_y,q_x,q_y\right)}{\mathrm{e}^{E^{(0)}_{q_x,q_y,\lambda}} - 1} \\ &= -ak\left[\pi\tau^{3/2} Z_{3/2}\left(\frac{2\lambda-1}{4\pi\tau}\right) - \frac{27}{16}\pi^2\tau^{5/2} Z_{5/2}\left(\frac{2\lambda-1}{4\pi\tau}\right)\right] \\ &\quad - (ak)^2\left[\frac{3}{4}\pi\tau^2 Z_2\left(\frac{2\lambda-1}{4\pi\tau}\right) - \frac{19}{16}\pi^2\tau^3 Z_3\left(\frac{2\lambda-1}{4\pi\tau}\right)\right] \\ &\quad + (ak)^3\left[\frac{1}{8}\pi\tau^{3/2} Z_{3/2}\left(\frac{2\lambda-1}{4\pi\tau}\right) - \frac{95}{768}\pi^2\tau^{5/2} Z_{5/2}\left(\frac{2\lambda-1}{4\pi\tau}\right)\right] + O\left(\tau^{7/2} Z\right)\end{aligned} \tag{2.14}$$

where

$$E^{(0)}_{k_x,k_y,\lambda} = 2I\left(\sin^2\frac{ak_x}{2} + \sin^2\frac{ak_y}{2} + \sin^2\frac{\lambda\pi}{6}\right); \quad \lambda = 1, 2 \tag{2.15}$$

and

$$M\left(k_x, k_y, q_x, q_y\right) = \cos ak_x + \cos ak_y + \cos aq_x + \cos aq_y - 2 - \cos a\left(k_x - q_x\right) - \cos a\left(k_y - q_y\right). \tag{2.16}$$

It can be easily shown that all Eqs. (2.9)–(2.11) after the substitution

$$\alpha_{n_z,m_z} = \sum_{\lambda=1}^{2} a_{m_z,\lambda} S_{n_z,\lambda}, \tag{2.17}$$

where

$$\begin{aligned} S_{n_z,\lambda} &= \sin\left(n_z + 1\right)\varphi_\lambda + Y_\lambda \sin n_z\varphi_\lambda; \\ Y_\lambda &= -1 + \left[\left(1 + \cos\varphi^{(0)}_\lambda\right)\Delta F + 3\Delta\Phi\right] \end{aligned} \tag{2.18}$$

reduce to the unique one

$$\sum_{\lambda=1}^{2}\left(2J\cos\varphi_\lambda + \rho\right) a_{m_z,\lambda}\left(k_x, k_y, \omega\right) S_{n_z,\lambda} = \frac{i\hbar}{2\pi}\delta_{n_z,m_z}; \quad n_z, m_z \in \{0, 1, 2\} \tag{2.19}$$

if the parameters φ_λ satisfy the following equation:

$$\frac{1}{2}\sin 3\varphi_\lambda - \sin 3\varphi_\lambda\cos\varphi_\lambda + \sin 2\varphi_\lambda\cos\varphi_\lambda - \frac{1}{2}\sin\varphi_\lambda + \frac{5\sqrt{3}}{8}\Delta F + \frac{3\sqrt{3}}{2}\Delta\Phi = 0. \tag{2.20}$$

The Kronecker symbol in (2.19) will be taken in the following form:

$$\delta_{n_z,m_z} = \sum_{\lambda=1}^{2} S_{n_z,\lambda} T_{m_z,\lambda}. \tag{2.21}$$

This system of algebraic equations enables us to calculate all introduced functions $T_{m_z,\lambda}$.

Substituting (2.21) in (2.19) and taking that

$$a_{m_z,\lambda}\left(k_x, k_y, \omega\right) = b_{k_x,k_y,\lambda}(\omega)\, T_{m_z,\lambda} \tag{2.22}$$

we reduce (2.19) into

$$b_{k_x,k_y,\lambda}(\omega) = \frac{i}{2\pi}\frac{1 + 2\langle B^+B\rangle^{(0)}}{\omega - \omega^{(1)}_{k_x,k_y,\lambda}}, \tag{2.23}$$

where

$$\begin{aligned} \omega_{k_x,k_y,\lambda} &= \frac{1}{\hbar}E^{(1)}_{k_x,k_y,\lambda}; \\ E^{(1)}_{k_x,k_y,\lambda} &= 2I\left(\sin^2\frac{ak_x}{2} + \sin^2\frac{ak_y}{2} + \sin^2\frac{\varphi_\lambda}{2}\right) - \frac{1}{2}IF_1 - \frac{1}{2}IF_2\left(3 - 4\cos\frac{\lambda\pi}{3}\right) + 2I\Phi_2. \end{aligned} \tag{2.24}$$

Combining (2.8), (2.17), (2.22), and inversing Fourier's time-frequency transformation of the Green function (2.8), we obtain:

$$G_{n_x,n_y,n_z;m_x,m_y,m_z}(\omega) = \frac{i\left(1 + 2\langle B^+B\rangle^{(0)}\right)}{2\pi}\frac{1}{N_xN_y}\cdot\sum_{k_x,k_y}\sum_{\lambda=1}^{2} \mathrm{e}^{iak_x(n_x-m_x)+iak_y\left(n_y-m_y\right)}\cdot\frac{S_{n_z,\lambda}T_{m_z,\lambda}}{\omega - \omega^{(0)}_{k_x,k_y,\lambda}} \tag{2.25}$$

in this formula the unknown functions $T_{m_z,\lambda}$ figure. They can be found from (2.21) putting $\delta_{n_z,m_z} = 1$ for $n_z = m_z$ and $\delta_{n_z,m_z} = 0$ for $n_z \neq m_z$.

Having formula (2.25), we can find the spectral intensity, correlation function and boson concentrations in the first approximation. We shall quote the first order boson concentration for subfilm (0, 1), only:

$$\langle B^+B\rangle^{(1)}_{01} = \left(1 + 2\langle B^+B\rangle^{(0)}\right)\frac{1}{N_xN_y}\cdot\sum_{k_x,k_y}\sum_{\lambda=1}^{2}\frac{S^{01}_{n_z,\lambda}T^{01}_{n_z,\lambda}}{\mathrm{e}^{\frac{E^{(1)}_{k_x,k_y,\lambda}}{\Theta}} - 1}. \tag{2.26}$$

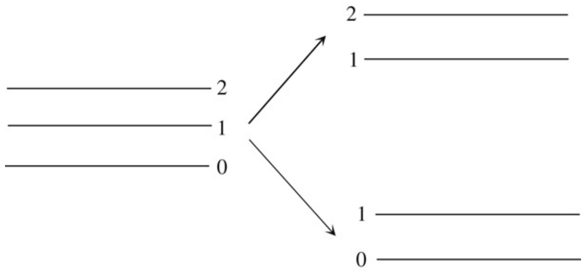

Fig. 1. Schematic presentation of autoreduction of three layer film into two equivalent two layer films.

Now we shall determine the ordering parameter and estimate the remaining order of magnitude. The expression for σ is given by:

$$\sigma = 1 - 2\langle P^+P\rangle = 1 - 2\langle B^+B\rangle^{(1)} + 4\left[\langle B^+B\rangle^{(0)}\right]^2 . \tag{2.27}$$

Taking into account (2.26), we can write down:

$$\sigma = 1 - \frac{2}{N_x N_y}\sum_{k_x,k_y}\sum_{\lambda=1}^{2}\frac{S_{n_z,\lambda}T_{n_z,\lambda}}{e^{\frac{E^{(1)}_{k_x,k_y,\lambda}}{\Theta}} - 1}. \tag{2.28}$$

From formula (2.28) the term proportional $\langle B^+B\rangle^{(0)}\left(\langle B^+B\rangle^{(1)} - \langle B^+B\rangle^{(0)}\right)$ was rejected since it is extremely small.

The fact that Eq. (2.20) has only two real solutions in interval $(0, \pi)$ is of great importance. Since the number of film layers has three values, the transformation from configuration space to momentum space is not isomorphic. It is of the type $3 \to 2$ and this means that the autoreduction takes place in the film [26–30]. The spin wave excitations exist in two layers only. It can be $(0, 1)$ and $(1, 2)$ sets of layers. The set $(0, 2)$ is incompatible with the nearest neighbour approximation which was applied. Geometrically, it is clear that sets $(0, 1)$ and $(1, 2)$ are completely equivalent (see Fig. 1).

It means that it is enough to examine magnetization in one of the mentioned subfilms. It was done for subfilm $(0, 1)$ and it was found that magnetizations of layers are approximately given by:

$$\sigma\,(n_z = 0) = \sigma_h\,(n_z = 0) + \sigma_a\,(n_z = 0) \tag{2.29}$$

where

$$\sigma_h\,(n_z = 0) = 1 - 2\tau Z_1\left(\frac{1}{4\pi\tau}\right) \tag{2.30}$$

and

$$\sigma_a\,(n_z = 0) = a\tau \tag{2.31}$$

where a is

$$a = \frac{1}{8\pi}Z_0\left(\frac{1}{4\pi\tau}\right)Z_1\left(\frac{1}{4\pi\tau}\right) - \frac{9}{8\pi}Z_0\left(\frac{1}{4\pi\tau}\right)Z_1\left(\frac{3}{4\pi\tau}\right). \tag{2.32}$$

The magnetization for layer $n_z = 1$ is given by

$$\sigma\,(n_z = 1) = \sigma_h\,(n_z = 1) + \sigma_a(n_z = 1) \tag{2.33}$$

where

$$\sigma_h\,(n_z = 1) = 1 - 2\tau Z_1\left(\frac{3}{4\pi\tau}\right) \tag{2.34}$$

and

$$\sigma_a\,(n_z = 1) \approx b\tau \tag{2.35}$$

where

$$b = \frac{11}{8\pi}Z_0\left(\frac{3}{4\pi\tau}\right)Z_1\left(\frac{1}{4\pi\tau}\right) + \frac{61}{8\pi}Z_0\left(\frac{3}{4\pi\tau}\right)Z_1\left(\frac{3}{4\pi\tau}\right). \tag{2.36}$$

In formulae (2.31)–(2.34) only the leading terms of magnetization are quoted.

3. Conclusion

Concluding this analysis, we would like to emphasize the following:

1. Magnetizations of layers 0 and 1 differ. It is the consequence of broken symmetry.
2. The temperature corrections are exponentially small due to the presence of zeta functions $Z_p\left(\frac{2\lambda-1}{4\pi\tau}\right)$.
3. The autoreduction of the $n \to n-1$ type takes place in the film reducing the three layer ferromagnet into two layer one.
4. In connection with experimental fact that Bose condensation takes place of spin waves in ferromagnet film YIG, our intention is to examine theoretically the behaviour of thin magnetic films in extremely strong magnetic fields.

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